Differentiation Rules
(AP Calculus AB)

$$y' = f'(x) = \frac{d[f(x)]}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Defining Continuity at a Point

$$\lim_{x \to a} f(x) = f(a)$$

$$\exists f'(a) \Rightarrow f(x)\ is\ continuous\ at\ \left(a, f(a)\right)$$

Intermediate Value Theorm (IVT)

$$f: A \to B,\ f(x) \in C([a,b])$$

$$\left(f(a) \le N \le f(b)\right) \cup \left(f(b) \le N \le f(a)\right)$$

$$\exists c \in (a,b) \Rightarrow f(c) = N$$

Squeeze Theorem

$$If\ \ (h(x) \le f(x) \le g(x)) \cap \left(\lim_{x \to a} h(x) = \lim_{x \to a} g(x) = L\right)$$

$$\Rightarrow \lim_{x \to a} f(x) = L$$

L'Hopital's Rule

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0}\ \text{or}\ \frac{\infty}{\infty} \xrightarrow{\text{LH Rule}} \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

$\frac{d[c]}{dx} = 0$	$\frac{d[u^n]}{dx} = nu^{n-1}u'$	$[c_1u \pm c_2v]' = c_1u' \pm c_2v'$

Product Rule $\Rightarrow$ $[uv]' = u'v + uv'$

Quotient Rule $\Rightarrow$ $\left[\frac{u}{v}\right]' = \frac{u'v - uv'}{v^2}$

Chain Rule

$$(f o g)'(u) = [f(g(u))]' = f'\left(g(u)\right)g'(u)u'$$

$[e^u]' = e^u u'$	$[b^u]' = (\ln b)b^u u'$
$[\ln u]' = \frac{u'}{u}$	$[\log_b u]' = \frac{u'}{(\ln b)u}$

$[\sin u]' = (\cos u)u'$ $[\cos u]' = -(\sin u)u'$	$[\tan u]' = (\sec^2 u)u'$

$[\cot u]' = -(\csc^2 u)u'$	$[\sec u]' = (\sec u \cdot \tan u)u'$	$[\csc u]' = -(\csc u \cdot \cot u)u'$

Derivative of the Inverse

$$f: A \to B, f \in C'(A), f'(x) \neq 0 \Rightarrow \exists f^{-1}: B \to A$$

$$(f^{-1})'(y) = \frac{1}{f'(f^{-1}(y))} = \frac{1}{f'(x)},\ \forall y \in B$$

Tangent line Equation

$$y - f(a) = f'(a)(x - a)$$

Linearization at $(a, f(a))$

$$L(x) = f(a) + f'(a)(x - a)$$

$[\sin^{-1} u]' = \frac{u'}{\sqrt{1 - u^2}}$	$[\tan^{-1} u]' = \frac{u'}{1 + u^2}$	$[\sec^{-1} u]' = \frac{u'}{\lvert u \rvert \sqrt{u^2 - 1}}$
$[\cos^{-1} u]' = -[\sin^{-1} u]'$	$[\cot^{-1} u]' = -[\tan^{-1} u]'$	$[\csc^{-1} u]' = -[\sec^{-1} u]'$

Newton's Method: $f(x) \approx 0$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, n = 0, 1, \ldots$$

Critical Point: $x = c \Rightarrow f'(c) = 0 \cup f'(c) = \text{DNE}$

$$f'(c) = 0 \cup f'(c) = \text{DNE} \begin{cases} f''(c) > 0 \Rightarrow local\ minimum\ (Concave\ Up) \\ f''(c) < 0 \Rightarrow local\ maximum\ (Concave\ Down) \\ f''(c) = 0 \Rightarrow inconclusive\ (Inflection\ point) \end{cases}$$

Mean Value Theorem (MVT)

$$f: A \to B, f(x) \in C([a,b]) \cap C'\left((a,b)\right)$$

$$\exists c \in (a,b) \Rightarrow f'(c) = \frac{f(b) - f(a)}{b - a}$$

Rolle's Theorem: If $f(a) = f(b)$

$$f: A \to B, f(x) \in C([a,b]) \cap C'\left((a,b)\right)$$

$$\exists c \in (a,b) \Rightarrow f'(c) = 0$$

Position	Vehicle at Rest	Total Displacement
$s(t), \quad t_0 \le t \le t_n$	$t_i \in \{t_1, .., t_k\} \Rightarrow v(t_i) = 0$	$s(t_n) - s(t_0)$

$$\text{Total Distance Traveled} = \sum_{i=0}^{n-1} \lvert s(t_{i+1}) - s(t_i) \rvert$$

Average Velocity ($\overline{v}$)

$$\overline{v} = \frac{s(b) - s(a)}{b - a}$$

$v(t) = s'(t)$	$a(t) = v'(t)$

Speed $= \lvert v(t) \rvert$

Moving Away	Moving Towards	Speeding	Slowing down
$s(t) \cdot v(t) > 0$	$s(t) \cdot v(t) < 0$	$v(t) \cdot a(t) > 0$	$v(t) \cdot a(t) < 0$