Logarithmic Properties

$(b > 0 \ \ and \ b \neq 1)$

$$f(x) = b^x \iff f^{-1}(x) = log_b x$$

$y = b^x \iff x = \log_b y$	$e^E = N \iff \ln N = E$
$\log_b b = 1$	$\ln e = 1$
$\log_b x^n = n\log_b x$	$\log_b a = \frac{1}{\log_a b}$
$b^{\log_b N} = N$	$\log_b \frac{1}{a} = -\log_b a$
$\log_b x + \log_b y = \log_b xy$	$\log_b x - \log_b y = \log_b \frac{x}{y}$

$$\log_b a = \frac{\log(a)}{\log(b)} = \frac{\ln(a)}{\ln(b)}$$

$$\log_{a^m}(a^n) = \frac{n}{m}, \quad m \neq 0$$

$$\log_{\left(\frac{1}{a}\right)} b = -\log_a b$$

$$\log_a b \cdot \log_b c = \log_a c$$

Logarithmic Properties

$(b > 0 \text{ and } b \neq 1)$

$$f(x) = b^x \quad \Leftrightarrow \quad f^{-1}(x) = log_b X$$

Properties	Examples	
1. $b^x = y \Leftrightarrow x = \log_b y$	$2^3 = 8 \Leftrightarrow \log_2 8 = 3$ $b^0 = 1 \Leftrightarrow \log_b 1 = 0$	
2. $e^x = y \Leftrightarrow x = \ln y$	$e^1 = e \Leftrightarrow \ln e = 1$	
3. $\log_b b = 1$	$\ln e = 1$	$\log_2 2 = 1$
4. $b^{\log_b N} = N$	$2^{\log_2 8} = 8$	$e^{\ln 7} = 7$
5. $\log_b X + \log_b Y = \log_b XY$	$\log_4 2 + \log_4 5 = \log_4(2 \times 5) = \log_4(10)$ $\ln 4 + \ln 6 = \ln(4 \times 6) = \ln(24)$	
6. $\log_b X - \log_b Y = \log_b \frac{X}{Y}$	$\log_5 10 - \log_5 2 = \log_5\left(\frac{10}{2}\right) = \log_5(5)$	
7. $\log_b \frac{1}{a} = -\log_b a$	$\log_2 \frac{1}{8} = -\log_2 8 = -3$ $\log_8 \frac{1}{2} = -\log_8 2 = -\frac{1}{3}$	
8. $\log_b X^n = n \log_b X$	$\log_6 7^2 = 2\log_6 7$	
9. $\log_b a = \frac{\log(a)}{\log(b)} = \frac{\ln(a)}{\ln(b)}$	$\log_4 5 = \frac{\log(5)}{\log(4)} = \frac{\ln(5)}{\ln(4)}$	
10. $\log_b a = \frac{1}{\log_a b}$	$\log_8 2 = \frac{1}{\log_2 8} = \frac{1}{3}$	
11. $\log_{a^m}(a^n) = \frac{n}{m}, \quad m \neq 0$	$\log_{2^3}(2^5) = \frac{5}{3}$	
12. $\log_{\left(\frac{1}{a}\right)} b = -\log_a b$	$\log_{\left(\frac{1}{2}\right)} 8 = -\log_2 8 = -3$	
13 $\log_a b \cdot \log_b c = \log_a c$	$\log_2 16 \cdot \log_{16} 64 = \log_2 64 = 6$	