Advanced Algebra Formula Sheet

Quadratic Function ($a \neq 0$)

$$f(x) = ax^2 + bx + c = a(x-h)^2 + k$$
$$\Delta = discriminat = b^2 - 4ac$$
$$(h,k) = \left(-\frac{b}{2a}, f(-\frac{b}{2a})\right) = \left(-\frac{b}{2a}, \frac{-\Delta}{4a}\right)$$

Factoring

$$a^2 - b^2 = (a-b)(a+b)$$
$$a^{2n} - b^{2n} = (a^n - b^n)(a^n + b^n)$$
$$a^4 + 4b^4 = (a^2 + 2ab + 2b^2)(a^2 - 2ab + 2b^2)$$
$$a^2 \pm 2ab + b^2 = (a \pm b)^2$$
$$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$$

Absolute Value

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

n^{th} root of x^n

$$\sqrt[n]{x^n} = \begin{cases} |x| & if\ n\ is\ even \\ x & if\ n\ is\ odd \end{cases}$$

Quadratic Formula

$$If\ \ ax^2 + bx + c = 0,\ \ a \neq 0,$$
$$\Rightarrow\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{\Delta}}{2a}$$

Midpoint (M) & Distance (d)

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Average Rate of Change

$$\text{AROC} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Arithmetic Series

$$f(1) = a_1, \quad f(n) = f(n-1) + d, n \geq 2$$
$$a_n = a_1 + (n-1)d, \quad n \geq 1$$
$$a_n = a_k + (n-k)d, \quad n \geq k$$
$$S_n = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}[2a_1 + (n-1)d]$$

Geometric Series

$$f(1) = a_1, \quad f(n) = r \cdot f(n-1), \ n \geq 2$$
$$a_n = a_1 r^{n-1}, \quad n \geq 1$$
$$a_n = a_k r^{n-k}, \quad n \geq k$$
$$S_n = \frac{a_1(1-r^n)}{1-r} \qquad S_\infty = \frac{a_1}{1-r}, |r| < 1$$

Period(P): $f(t+P) = f(t)$

E. g: $f(t) = Asin(\omega t - \phi) + C$
$f(t) = Acos(\omega t - \phi) + C$
$$P = \frac{2\pi}{\omega}\ ;\ f = \frac{\omega}{2\pi};$$
$$Phase\ Shift = \frac{\phi}{\omega}$$

Factorial ($n_1\ alike, \ldots$)

$$\frac{n!}{(n_1)!\,(n_2)!\ldots(n_k)!}$$

Combination

$${}^nC_r = \binom{n}{r} = \frac{n!}{(n-r)!\,r!}$$

Permutation

$${}^nP_r = \frac{n!}{(n-r)!}$$

$\pi \approx 3.14$

$$\pi = \frac{C}{d}$$

$e \approx 2.72$

$$e = \lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n$$

Complex Numbers

$z = a + ib$ *where* $i = \sqrt{-1} \Rightarrow i^2 = -1$
$r = |z| = \sqrt{a^2 + b^2}$
$z = x + iy$ $(Rectangular\ form)$
$z = r(\cos\theta + i\sin\theta)$ $(Polar\ form)$
$z = re^{i\theta}$ $(Euler\ \ form)$

Inverse Function

$$(f \circ f^{-1})(x) = f(f^{-1}(x)) = x$$
$$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = x$$

Pythagorean Theorem

$$a^2 + b^2 = c^2$$

Doubling − Time

$$Q = Q_0 \cdot 2^{t/T_{double}}$$

Half − Life Time

$$Q = Q_0 \cdot \left(\frac{1}{2}\right)^{t/T_{half}}$$

Continuous Compounding

$$A = \lim_{n\to\infty} P\left(1 + \frac{r}{n}\right)^{nt} = Pe^{rt}$$

$180^\circ = \pi$ radians

$$\sin\theta = \frac{O}{H}$$
$$\cos\theta = \frac{A}{H}$$
$$\tan\theta = \frac{O}{A}$$

Pascal Triangle

$$\binom{0}{0}$$
$$\binom{1}{0}\binom{1}{1}$$
$$\binom{2}{0}\binom{2}{1}\binom{2}{2}$$
$$\binom{3}{0}\binom{3}{1}\binom{3}{2}\binom{3}{3}$$
$$\binom{4}{0}\binom{4}{1}\binom{4}{2}\binom{4}{3}\binom{4}{4}$$

Binomial Theorem

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^2b + \cdots \binom{n}{r-1}a^{n-r+1}b^{r-1} + \binom{n}{r}a^{n-r}b^r + \cdots + \binom{n}{n}b^n$$

E. g: $(a+b)^4 = \binom{4}{0}a^4b^0 + \binom{4}{1}a^3b^1 + \binom{4}{2}a^2b^2 + \binom{4}{3}a^1b^3 + \binom{4}{4}a^0b^4$
$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

Exponential Properties

$$b^0 = 1 \qquad b^m b^n = b^{m+n}$$
$$(ab)^n = a^n b^n \qquad (b^m)^n = b^{mn}$$
$$b^{-n} = \frac{1}{b^n} \qquad \frac{b^m}{b^n} = b^{m-n}$$
$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \qquad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$
$$b^{\frac{1}{n}} = \sqrt[n]{b} \qquad b^{\frac{m}{n}} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$$

Linear Equation

$$y = mx + b$$
$$y - y_1 = m(x - x_1)$$
$$Ax + By = C$$
$$\frac{x}{a} + \frac{y}{b} = 1$$
$$ax + by + c = 0$$

Circle

$$(x-h)^2 + (y-k)^2 = r^2$$

Ellipse

↔: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
↕: $\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$

Parabola

→ ←: $(x-h) = \frac{1}{4p}(y-k)^2$
↑↓: $(y-k) = \frac{1}{4p}(x-h)^2$

Hyperbola

↔: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
↕: $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

Special Triangles

$45^\circ : 45^\circ : 90^\circ\ \Delta$
$x : x : x\sqrt{2}$

$30^\circ : 60^\circ : 90^\circ\ \Delta$
$x : x\sqrt{3} : 2x$

Parallel & Perpendicular Lines

↕ $x = a$ ↔ $y = b$

↕↕ $m_1 = m_2$
$\perp\ m_1 \times m_2 = -1$

Logarithm Properties ($b \neq 0, b \neq 1$)

$$\log_b x + \log_b y = \log_b xy$$
$$y = b^x \iff x = \log_b y$$
$$\log_b x - \log_b y = \log_b\left(\frac{x}{y}\right)$$
$$\log_b b = 1 \qquad \ln e = 1 \qquad \log_b 1 = 0$$
$$\log_b a = \frac{1}{\log_a b}$$
$$\log_b x^n = n\log_b x$$
$$b^{\log_b N} = N$$
$$\log_b a = \frac{\log(a)}{\log(b)}$$
$$\log_b \frac{1}{a} = -\log_b a$$

$$d = rt \qquad s = r\theta \qquad \theta = \omega t \qquad v = r\omega$$

DMS: $1^\circ = 60'$ and $1' = 60''$