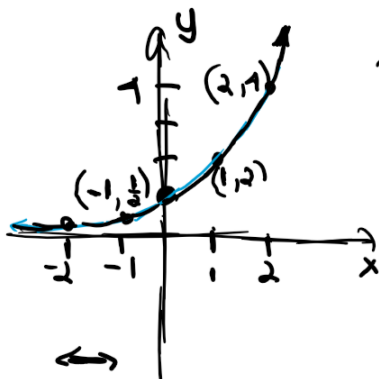


Graph $f(x) = 2^x$



$$f(x) = 2^x > 0$$

$$y > 0$$

x	$f(x)$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

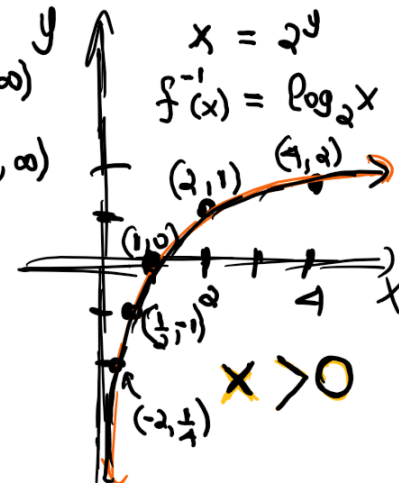
x	$f^{-1}(x)$
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2

$$y = 2^x \xrightarrow{f^{-1}} x = 2^y$$

$$\text{domain} = (0, \infty)$$

$$\text{Range} = (-\infty, \infty)$$

Graph $f^{-1}(x)$

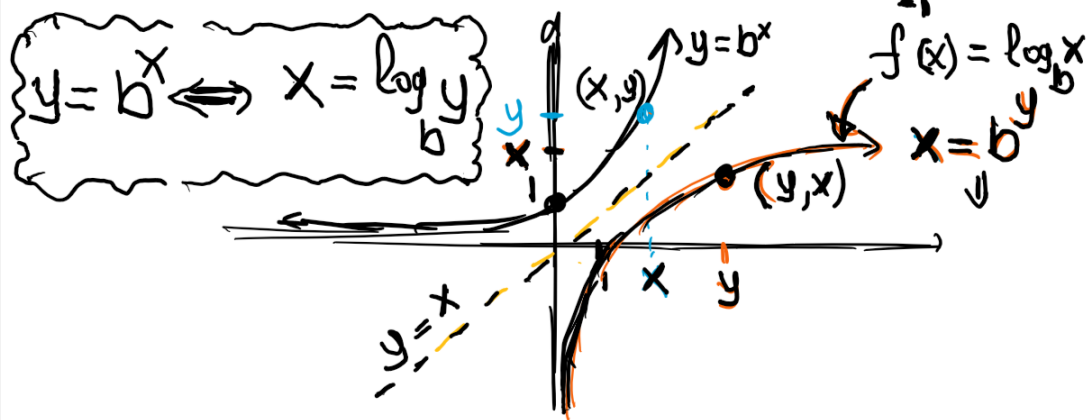


$$x = 2^y$$

$$f^{-1}(x) = \log_2 x$$

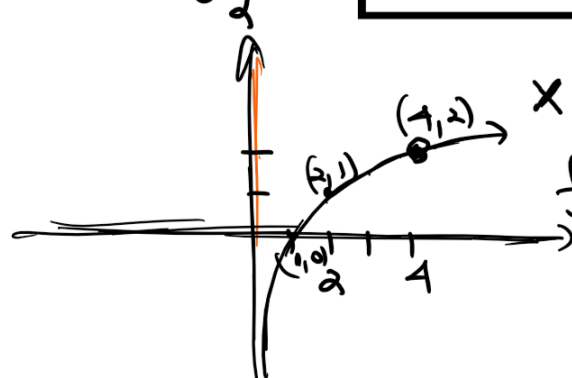
$$x > 0$$

Inverse function of $f(x) = b^x$ $b > 1$



$f(x)$ & $f^{-1}(x)$

$$f^{-1}(x) = \log_2 x$$



$$x = 2^y$$

$$f^{-1}(x) = \log_2 x$$

$$\text{domain} = (0, \infty)$$

$$\text{Range} = (-\infty, \infty)$$

$$\log_2 8 = \frac{\log 8}{\log 2} = 3$$

$$2^3 = 8 \Leftrightarrow \log 8 = 3$$

$$2^1 = 2 \Leftrightarrow \log_2 2 = 1$$

Properties

$$2^3 = 8 \Leftrightarrow \log_2 8 = 3$$

$$b^1 = b \Rightarrow \log_b b = 1$$

$$b^0 = 1 \Rightarrow \log_b 1 = 0$$

$$\ln e = 1$$

$$\ln 1 = 0$$

Convert $2^3 = 8$ to logarithmic form

$$2^3 = 8 \Rightarrow \log_2 8 = 3$$

Convert $2^{-3} = \frac{1}{8}$ to logarithmic form

$$2^{-3} = \frac{1}{8} \Rightarrow \log_2 \left(\frac{1}{8}\right) = -3$$

$$y = b^x \Leftrightarrow x = \log_b y$$

Solve $2^x = 16$

Method - 1

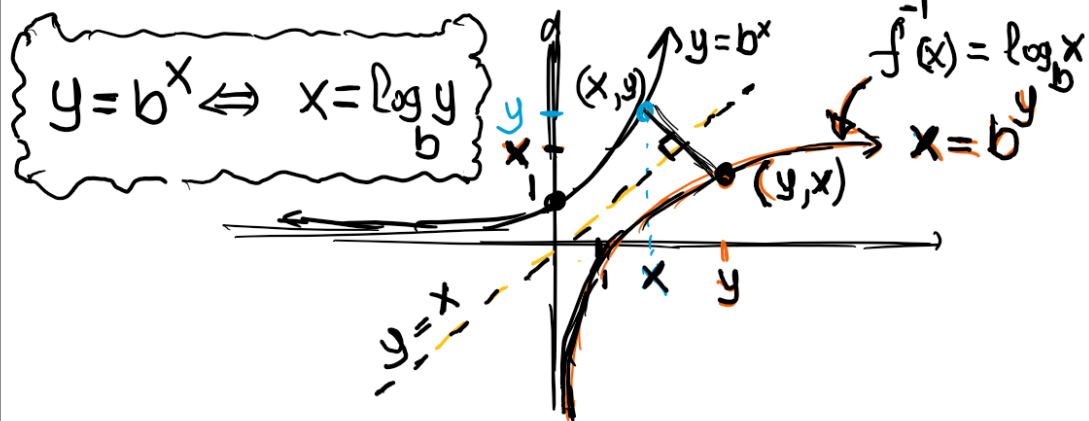
$$2^x = 16 = 2^4 \Rightarrow x = 4$$

Method - 2

$$2^x = 16 \Rightarrow x = \log_2 16 = \log_2 2^4 = 4 \log_2 2$$

$$x = 4$$

Inverse function of $f(x) = b^x, b > 1$



$$y = b^x \Leftrightarrow x = \log_b y$$

Convert $\log_2 2 = 1$ to exponential form

$$\log_2 2 = 1 \Leftrightarrow 2^1 = 2$$

Convert $\log_5 1 = 0$ to exponential form

$$\log_5 1 = 0 \Leftrightarrow 5^0 = 1$$

$$\log_b b = 1, \log_b 1 = 0$$

Solve $64^x = 16$

$$\log_b a = \frac{\log a}{\log b} = \frac{\ln a}{\ln b}$$

$$64^x = 16 \Rightarrow x = \log_{64} 16$$

$$x = \frac{\log_4 16}{\log_4 64} = \frac{\log_4 4^2}{\log_4 4^3} = \frac{2(\log_4 4)}{3(\log_4 4)} = \frac{2}{3}$$

$$\log_b x^n = n \log_b x$$

Simplify $\log_8 16 - \log_8 2$

$$\log_b x + \log_b y = \log_b (xy)$$

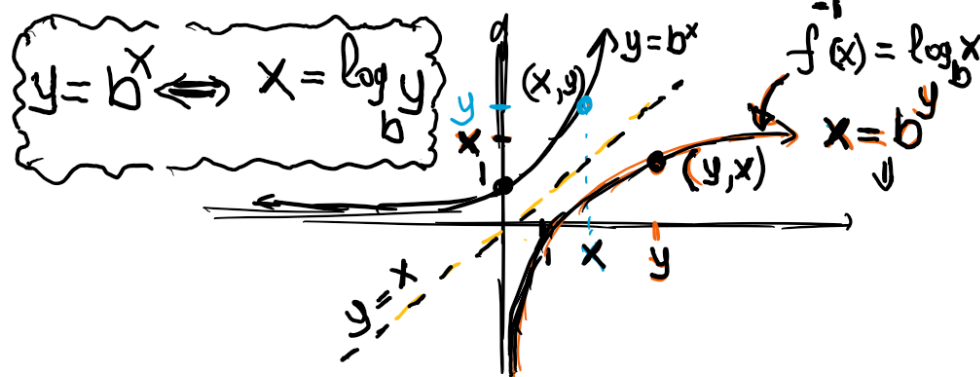
$$\begin{aligned} \log_8 16 - \log_8 2 \\ &= \log_8 (8 \times 2) - \log_8 2 \\ &= \log_8 8 + \log_8 2 - \log_8 2 \\ &= 1 \end{aligned}$$

Simplify $\log_8 16 - \log_8 2$

$$\log_b x - \log_b y = \log_b \left(\frac{x}{y}\right)$$

$$\begin{aligned} &= \log_8 16 - \log_8 2 \\ &= \log_8 \left(\frac{16}{2}\right) = \log_8 8 \\ &= 1 \end{aligned}$$

Logarithmic Properties



Simplify

$$\begin{aligned} \log_8 16 - \log_8 2 \\ &= \log_8 2^4 - \log_8 2 \\ &= 4 \log_8 2 - 1 \log_8 2 \\ &= 3 \log_8 2 \\ &= 3 \left(\frac{1}{3}\right) = 1 \end{aligned}$$

$$\begin{aligned} \sqrt[3]{8} &= 8^{1/3} = 2 \\ &\Downarrow \\ \log_8 2 &= \frac{1}{3} \end{aligned}$$

* Simplify

$$\log_8 16 - \log_8 2$$

$$\begin{aligned} x = \log_8 16 &\Rightarrow 8^x = 16 \Rightarrow 2^{3x} = 2^4 \Rightarrow x = \frac{4}{3} \\ y = \log_8 2 &\Rightarrow 8^y = 2 \Rightarrow 2^{3y} = 2^1 \Rightarrow y = \frac{1}{3} \\ \log_8 16 - \log_8 2 &= x - y = \frac{4}{3} - \frac{1}{3} = \frac{3}{3} = 1 \end{aligned}$$